

## 13.1 | Newton's Law of Universal Gravitation

### Learning Objectives

By the end of this section, you will be able to:

- List the significant milestones in the history of gravitation
- Calculate the gravitational force between two point masses
- Estimate the gravitational force between collections of mass

We first review the history of the study of gravitation, with emphasis on those phenomena that for thousands of years have inspired philosophers and scientists to search for an explanation. Then we examine the simplest form of Newton's law of universal gravitation and how to apply it.

### The History of Gravitation

The earliest philosophers wondered why objects naturally tend to fall toward the ground. Aristotle (384–322 BCE) believed that it was the nature of rocks to seek Earth and the nature of fire to seek the Heavens. Brahmagupta (598–665 CE) postulated that Earth was a sphere and that objects possessed a natural affinity for it, falling toward the center from wherever they were located.

The motions of the Sun, our Moon, and the planets have been studied for thousands of years as well. These motions were described with amazing accuracy by Ptolemy (90–168 CE), whose method of epicycles described the paths of the planets as circles within circles. However, there is little evidence that anyone connected the motion of astronomical bodies with the motion of objects falling to Earth—until the seventeenth century.

Nicolaus Copernicus (1473–1543) is generally credited as being the first to challenge Ptolemy's geocentric (Earth-centered) system and suggest a heliocentric system, in which the Sun is at the center of the solar system. This idea was supported by the incredibly precise naked-eye measurements of planetary motions by Tycho Brahe and their analysis by Johannes Kepler and Galileo Galilei. Kepler showed that the motion of each planet is an ellipse (the first of his three laws, discussed in **Kepler's Laws of Planetary Motion**), and Robert Hooke (the same Hooke who formulated Hooke's law for springs) intuitively suggested that these motions are due to the planets being attracted to the Sun. However, it was Isaac Newton who connected the acceleration of objects near Earth's surface with the centripetal acceleration of the Moon in its orbit about Earth.

Finally, in **Einstein's Theory of Gravity**, we look at the theory of general relativity proposed by Albert Einstein in 1916. His theory comes from a vastly different perspective, in which gravity is a manifestation of mass warping space and time. The consequences of his theory gave rise to many remarkable predictions, essentially all of which have been confirmed over the many decades following the publication of the theory (including the 2015 measurement of gravitational waves from the merger of two black holes).

### Newton's Law of Universal Gravitation

Newton noted that objects at Earth's surface (hence at a distance of  $R_E$  from the center of Earth) have an acceleration of  $g$ , but the Moon, at a distance of about  $60 R_E$ , has a centripetal acceleration about  $(60)^2$  times smaller than  $g$ . He could explain this by postulating that a force exists between any two objects, whose magnitude is given by the product of the two masses divided by the square of the distance between them. We now know that this inverse square law is ubiquitous in nature, a function of geometry for point sources. The strength of any source at a distance  $r$  is spread over the surface of a sphere centered about the mass. The surface area of that sphere is proportional to  $r^2$ . In later chapters, we see this same form in the electromagnetic force.

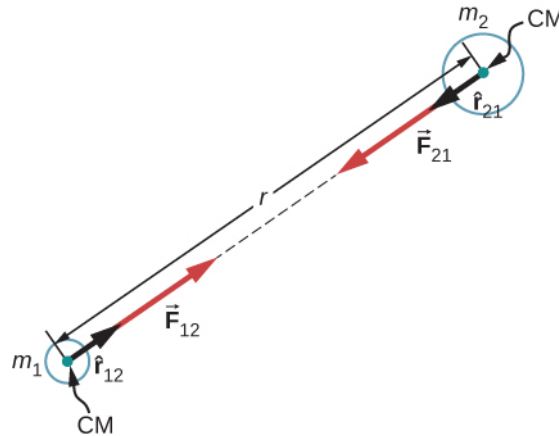
#### Newton's Law of Gravitation

Newton's law of gravitation can be expressed as

$$\vec{\mathbf{F}}_{12} = G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{12} \quad (13.1)$$

where  $\vec{\mathbf{F}}_{12}$  is the force on object 1 exerted by object 2 and  $\hat{\mathbf{r}}_{12}$  is a unit vector that points from object 1 toward object 2.

As shown in **Figure 13.2**, the  $\vec{\mathbf{F}}_{12}$  vector points from object 1 toward object 2, and hence represents an attractive force between the objects. The equal but opposite force  $\vec{\mathbf{F}}_{21}$  is the force on object 2 exerted by object 1.



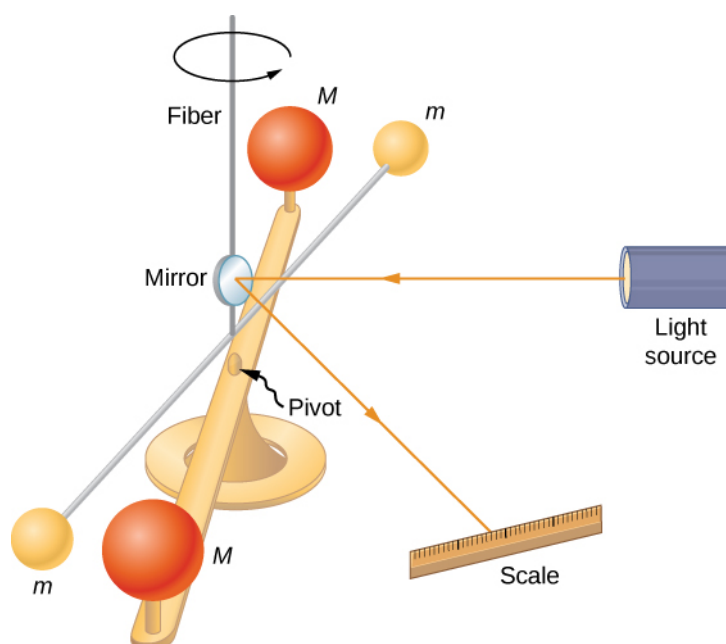
**Figure 13.2** Gravitational force acts along a line joining the centers of mass of two objects.

These equal but opposite forces reflect Newton's third law, which we discussed earlier. Note that strictly speaking, **Equation 13.1** applies to point masses—all the mass is located at one point. But it applies equally to any spherically symmetric objects, where  $r$  is the distance between the centers of mass of those objects. In many cases, it works reasonably well for nonsymmetrical objects, if their separation is large compared to their size, and we take  $r$  to be the distance between the center of mass of each body.

## The Cavendish Experiment

A century after Newton published his law of universal gravitation, Henry Cavendish determined the proportionality constant  $G$  by performing a painstaking experiment. He constructed a device similar to that shown in **Figure 13.3**, in which small masses are suspended from a wire. Once in equilibrium, two fixed, larger masses are placed symmetrically near the smaller ones. The gravitational attraction creates a torsion (twisting) in the supporting wire that can be measured.

The constant  $G$  is called the **universal gravitational constant** and Cavendish determined it to be  $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ . The word 'universal' indicates that scientists think that this constant applies to masses of any composition and that it is the same throughout the Universe. The value of  $G$  is an incredibly small number, showing that the force of gravity is very weak. The attraction between masses as small as our bodies, or even objects the size of skyscrapers, is incredibly small. For example, two 1.0-kg masses located 1.0 meter apart exert a force of  $6.7 \times 10^{-11} \text{ N}$  on each other. This is the weight of a typical grain of pollen.



**Figure 13.3** Cavendish used an apparatus similar to this to measure the gravitational attraction between two spheres ( $m$ ) suspended from a wire and two stationary spheres ( $M$ ). This is a common experiment performed in undergraduate laboratories, but it is quite challenging. Passing trucks outside the laboratory can create vibrations that overwhelm the gravitational forces.

Although gravity is the weakest of the four fundamental forces of nature, its attractive nature is what holds us to Earth, causes the planets to orbit the Sun and the Sun to orbit our galaxy, and binds galaxies into clusters, ranging from a few to millions. Gravity is the force that forms the Universe.

### Problem-Solving Strategy: Newton's Law of Gravitation

To determine the motion caused by the gravitational force, follow these steps:

1. Identify the two masses, one or both, for which you wish to find the gravitational force.
2. Draw a free-body diagram, sketching the force acting on each mass and indicating the distance between their centers of mass.
3. Apply Newton's second law of motion to each mass to determine how it will move.

## Example 13.1

### A Collision in Orbit

Consider two nearly spherical Soyuz payload vehicles, in orbit about Earth, each with mass 9000 kg and diameter 4.0 m. They are initially at rest relative to each other, 10.0 m from center to center. (As we will see in **Kepler's Laws of Planetary Motion**, both orbit Earth at the same speed and interact nearly the same as if they were isolated in deep space.) Determine the gravitational force between them and their initial acceleration. Estimate how long it takes for them to drift together, and how fast they are moving upon impact.

### Strategy

We use Newton's law of gravitation to determine the force between them and then use Newton's second law to find the acceleration of each. For the *estimate*, we assume this acceleration is constant, and we use the constant-acceleration equations from **Motion along a Straight Line** to find the time and speed of the collision.

**Solution**

The magnitude of the force is

$$|\vec{\mathbf{F}}_{12}| = F_{12} = G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \frac{(9000 \text{ kg})(9000 \text{ kg})}{(10 \text{ m})^2} = 5.4 \times 10^{-5} \text{ N}.$$

The initial acceleration of each payload is

$$a = \frac{F}{m} = \frac{5.4 \times 10^{-5} \text{ N}}{9000 \text{ kg}} = 6.0 \times 10^{-9} \text{ m/s}^2.$$

The vehicles are 4.0 m in diameter, so the vehicles move from 10.0 m to 4.0 m apart, or a distance of 3.0 m each. A similar calculation to that above, for when the vehicles are 4.0 m apart, yields an acceleration of  $3.8 \times 10^{-8} \text{ m/s}^2$ , and the average of these two values is  $2.2 \times 10^{-8} \text{ m/s}^2$ . If we assume a constant acceleration of this value and they start from rest, then the vehicles collide with speed given by

$$v^2 = v_0^2 + 2a(x - x_0), \text{ where } v_0 = 0,$$

so

$$v = \sqrt{2(2.2 \times 10^{-8} \text{ N})(3.0 \text{ m})} = 3.6 \times 10^{-4} \text{ m/s}.$$

We use  $v = v_0 + at$  to find  $t = v/a = 1.7 \times 10^4 \text{ s}$  or about 4.6 hours.

**Significance**

These calculations—including the initial force—are only estimates, as the vehicles are probably not spherically symmetrical. But you can see that the force is incredibly small. Astronauts must tether themselves when doing work outside even the massive International Space Station (ISS), as in **Figure 13.4**, because the gravitational attraction cannot save them from even the smallest push away from the station.



**Figure 13.4** This photo shows Ed White tethered to the Space Shuttle during a spacewalk. (credit: NASA)



**13.1 Check Your Understanding** What happens to force and acceleration as the vehicles fall together? What will our estimate of the velocity at a collision higher or lower than the speed actually be? And finally, what would happen if the masses were not identical? Would the force on each be the same or different? How about their accelerations?

The effect of gravity between two objects with masses on the order of these space vehicles is indeed small. Yet, the effect of gravity on you from Earth is significant enough that a fall into Earth of only a few feet can be dangerous. We examine the force of gravity near Earth's surface in the next section.

## Example 13.2

### Attraction between Galaxies

Find the acceleration of our galaxy, the Milky Way, due to the nearest comparably sized galaxy, the Andromeda galaxy (**Figure 13.5**). The approximate mass of each galaxy is 800 billion solar masses (a solar mass is the mass of our Sun), and they are separated by 2.5 million light-years. (Note that the mass of Andromeda is not so well known but is believed to be slightly larger than our galaxy.) Each galaxy has a diameter of roughly 100,000 light-years ( $1 \text{ light-year} = 9.5 \times 10^{15} \text{ m}$ ).



**Figure 13.5** Galaxies interact gravitationally over immense distances. The Andromeda galaxy is the nearest spiral galaxy to the Milky Way, and they will eventually collide. (credit: Boris Štromar)

### Strategy

As in the preceding example, we use Newton's law of gravitation to determine the force between them and then use Newton's second law to find the acceleration of the Milky Way. We can consider the galaxies to be point masses, since their sizes are about 25 times smaller than their separation. The mass of the Sun (see **Appendix D**) is  $2.0 \times 10^{30} \text{ kg}$  and a light-year is the distance light travels in one year,  $9.5 \times 10^{15} \text{ m}$ .

### Solution

The magnitude of the force is

$$F_{12} = G \frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2) \frac{[(800 \times 10^9)(2.0 \times 10^{30} \text{ kg})]^2}{[(2.5 \times 10^6)(9.5 \times 10^{15} \text{ m})]^2} = 3.0 \times 10^{29} \text{ N}.$$

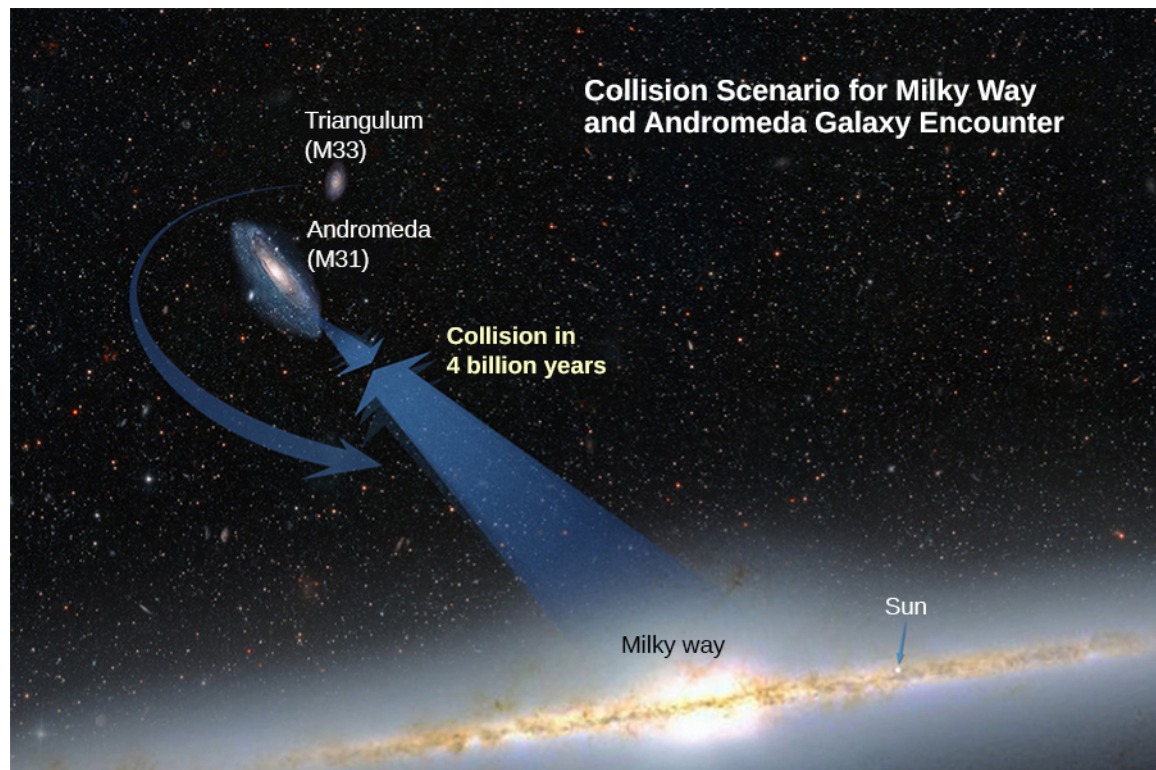
The acceleration of the Milky Way is

$$a = \frac{F}{m} = \frac{3.0 \times 10^{29} \text{ N}}{(800 \times 10^9)(2.0 \times 10^{30} \text{ kg})} = 1.9 \times 10^{-13} \text{ m/s}^2.$$



### Significance

Does this value of acceleration seem astoundingly small? If they start from rest, then they would accelerate directly toward each other, “colliding” at their center of mass. Let’s estimate the time for this to happen. The initial acceleration is  $\sim 10^{-13} \text{ m/s}^2$ , so using  $v = at$ , we see that it would take  $\sim 10^{13} \text{ s}$  for each galaxy to reach a speed of  $1.0 \text{ m/s}$ , and they would be only  $\sim 0.5 \times 10^{13} \text{ m}$  closer. That is nine orders of magnitude smaller than the initial distance between them. In reality, such motions are rarely simple. These two galaxies, along with about 50 other smaller galaxies, are all gravitationally bound into our local cluster. Our local cluster is gravitationally bound to other clusters in what is called a supercluster. All of this is part of the great cosmic dance that results from gravitation, as shown in **Figure 13.6**.



**Figure 13.6** Based on the results of this example, plus what astronomers have observed elsewhere in the Universe, our galaxy will collide with the Andromeda Galaxy in about 4 billion years. (credit: modification of work by NASA; ESA; A. Feild and R. van der Marel, STScI)

## 13.2 | Gravitation Near Earth's Surface

### Learning Objectives

By the end of this section, you will be able to:

- Explain the connection between the constants  $G$  and  $g$
- Determine the mass of an astronomical body from free-fall acceleration at its surface
- Describe how the value of  $g$  varies due to location and Earth's rotation

In this section, we observe how Newton’s law of gravitation applies at the surface of a planet and how it connects with what we learned earlier about free fall. We also examine the gravitational effects within spherical bodies.